Currents from hot spots

Markus Büttiker, Geneva

with

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Summer School "Energy harvesting at micro and nanoscales“,
Workshop "Energy harvesting: models and applications“, Erice, Italy, July 23-27, 2012

Topics

Introduction

Currents from noise in inversion symmetric potentials (Lecture of F. Marchesoni)

Currents from noise in inversion symmetric potentials (Lecture of Fabian Hartmann)

Coulomb coupled quantum dots

Currents in coupled quantum dots: Coulomb drag and current correlations

A minimal model in the Coulomb blockade regime

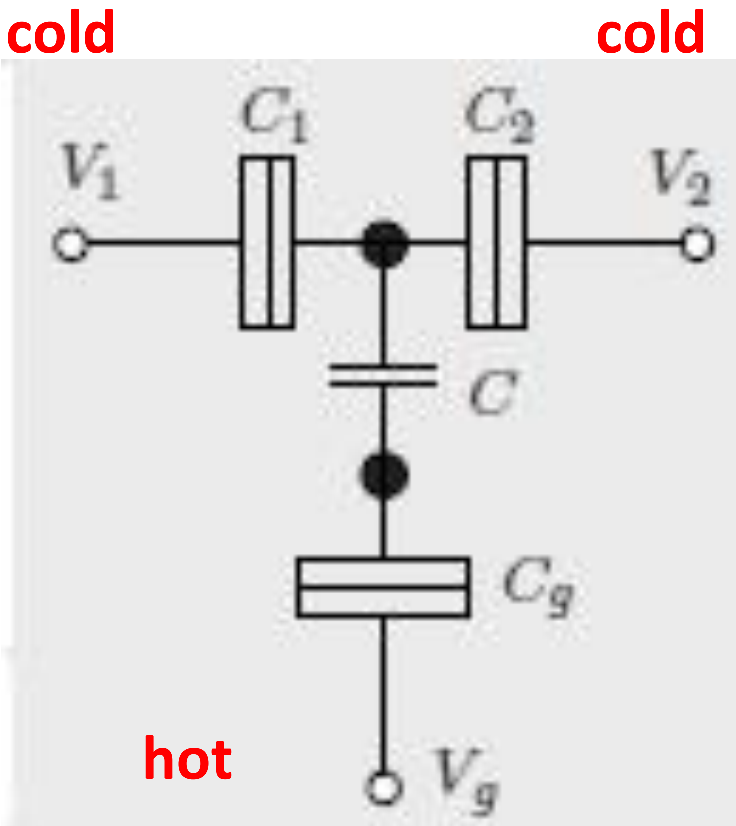
Open quantum dots: scaling with channel number

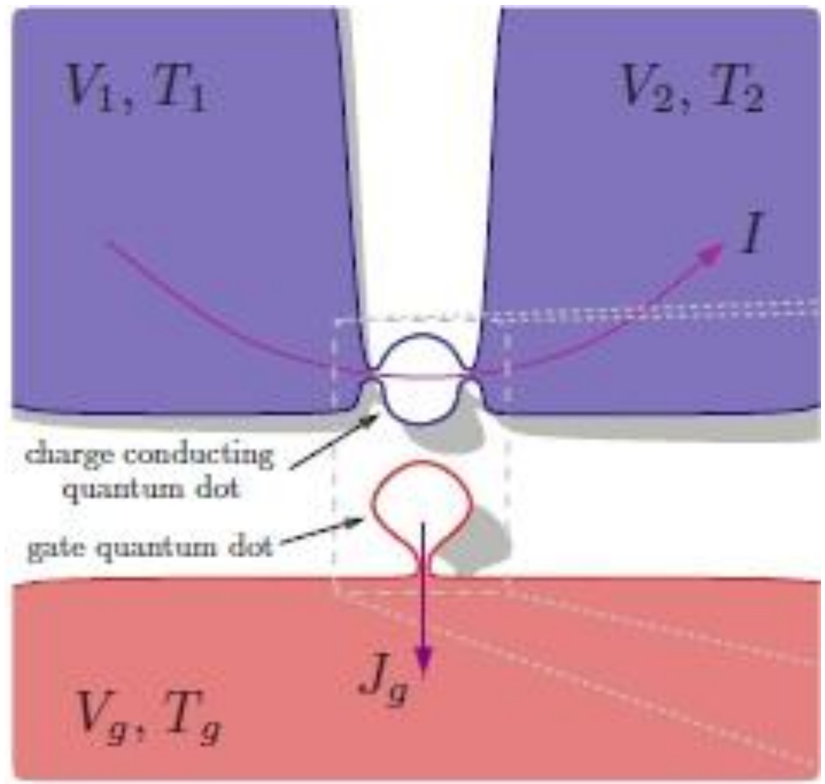
The measurement problem

Spintronic quantum dot

Magnon harvester (Workshop Lecture of Björn Sothmann)

Three-terminal thermoelectrics





Directional separation of heat and current flows

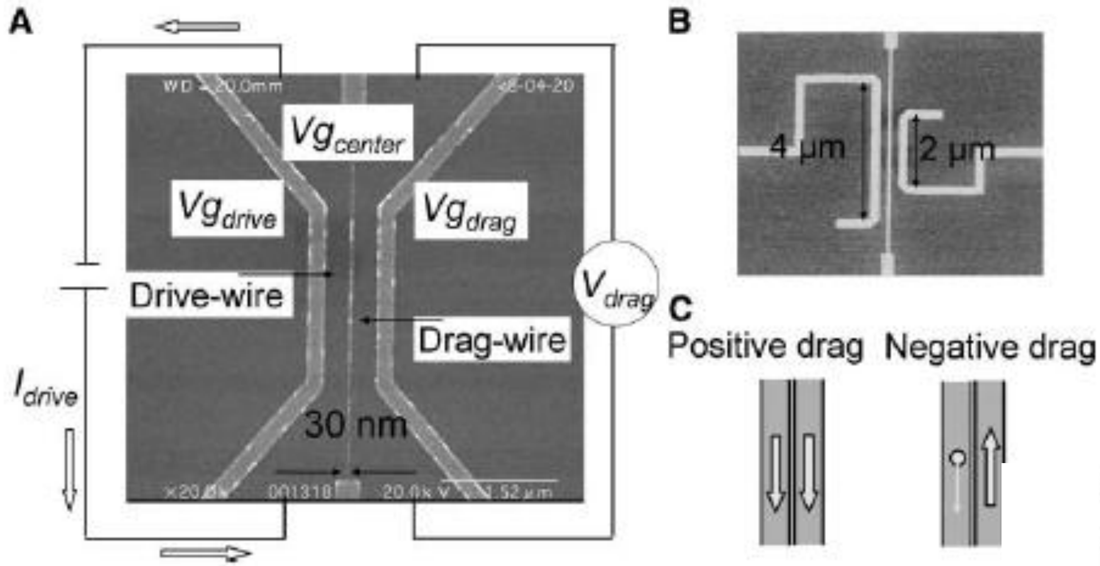
Separation of heat source and rectifier

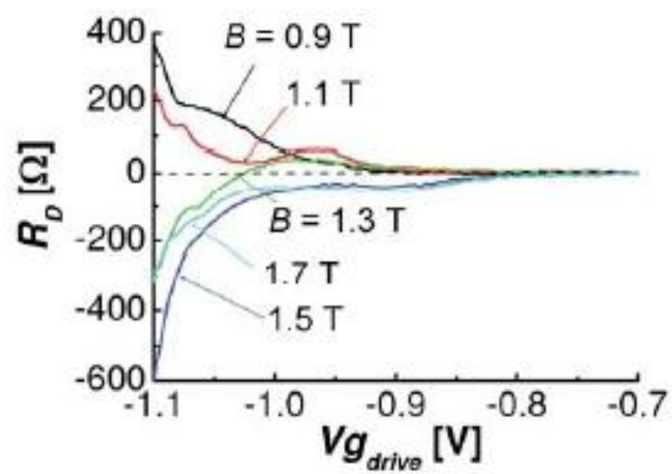
Coulomb coupled conductors

Coulomb drag

Example form the mesoscopic literature

M. Yamamoto et al. (Tarucha) , Science 313, 204 (2006).



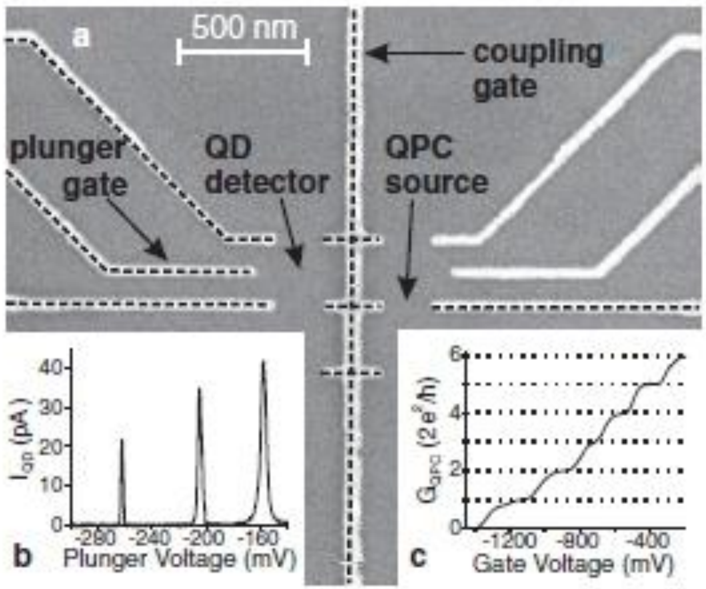


Drag resistance

$$R_D = V_{drag}/I_{drive}$$

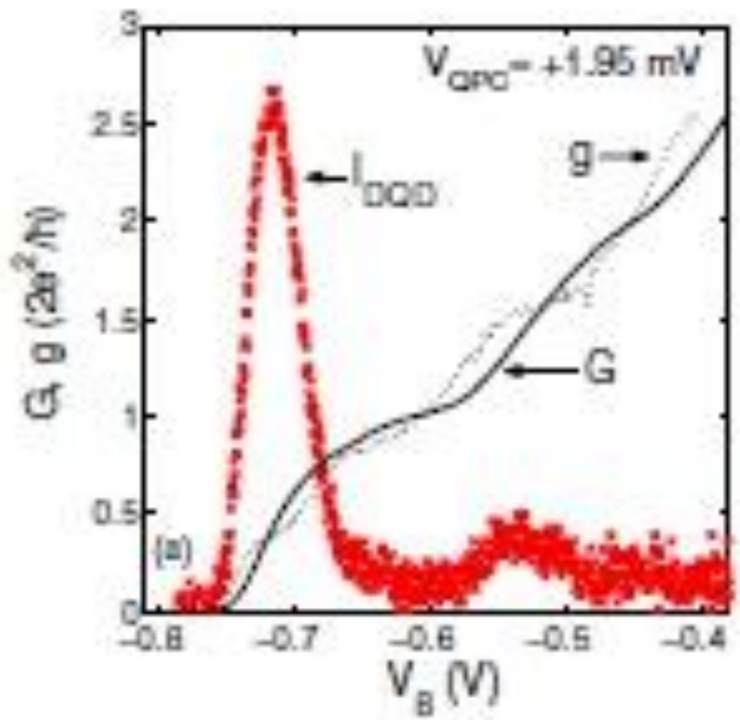
Source and Detector

E. Onac (Kouwenhoven) et al., PRL 96, 176601 (2006)

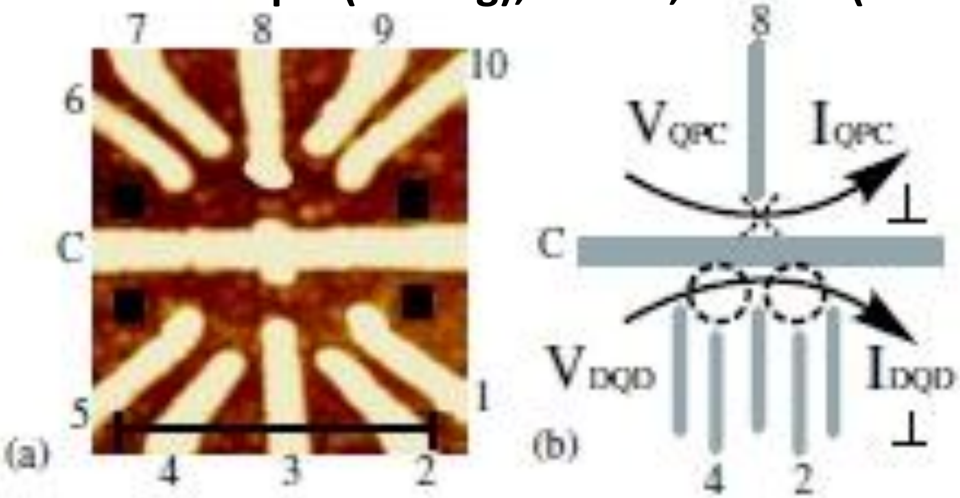


QD current modified by QPC noise

Phonons?

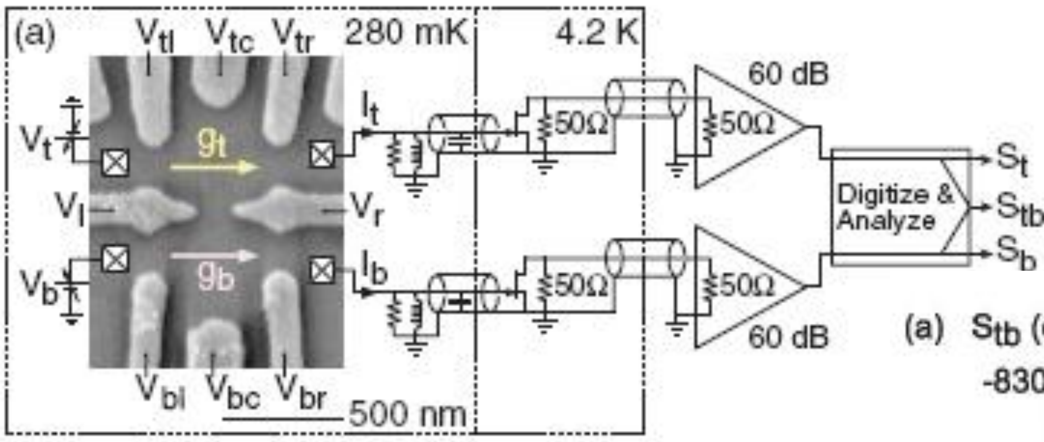


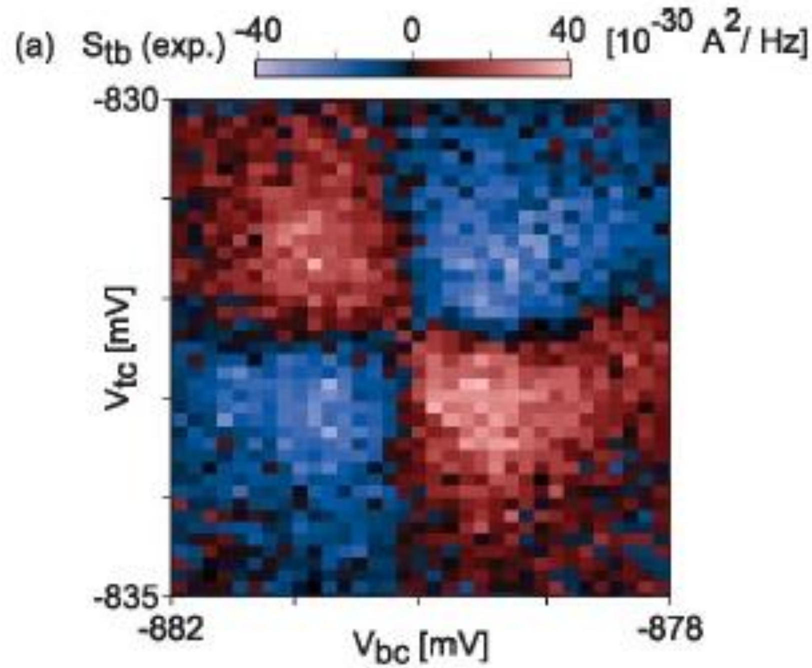
V. S. Kharpai (Ludwig), PRL 97, 176803 (2006)



Current correlation of quantum dots

T. D. McClure et al., PRL 98, 056801 (2007)





Current correlations

$$S_{12} = \int d\tau < \Delta I_1(\tau)\, \Delta I_2(0\,) >$$

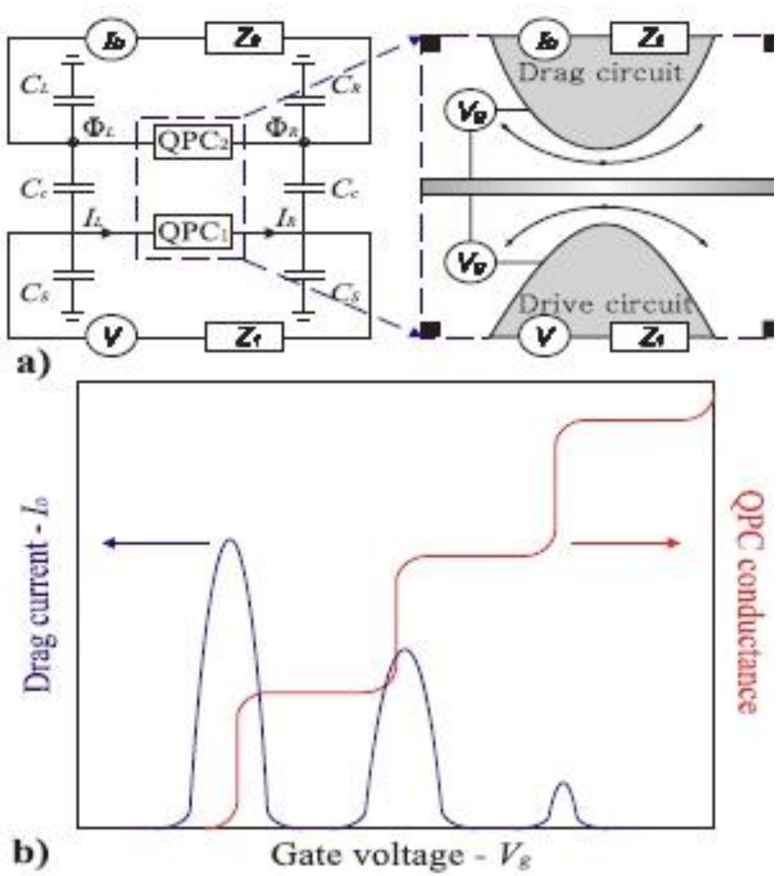
M. C. Goorden and M. Buttiker, PRL 99; 146801 (2007); PRB 77, 205323 (2008)

Noise induced transport

Noise induced transport

A. Levchenko and A. Kamenev, PRL 101, 216806 (2008)

Capacitively coupled quantum point contacts



External impedances

$$Z_1 << Z_2 << \frac{h}{e^2}$$

Linear drag (thermal noise?)

$$I_D(V) = \frac{h}{e^2}\frac{\alpha\pi^2}{6}\frac{(kT)^2}{\Delta_1\Delta_2}\frac{V}{cosh^2(eV_g/2\Delta_1)}$$

Coupling constant α

Non-linear drag (shot noise)

$$kT << eV << \Delta_1$$

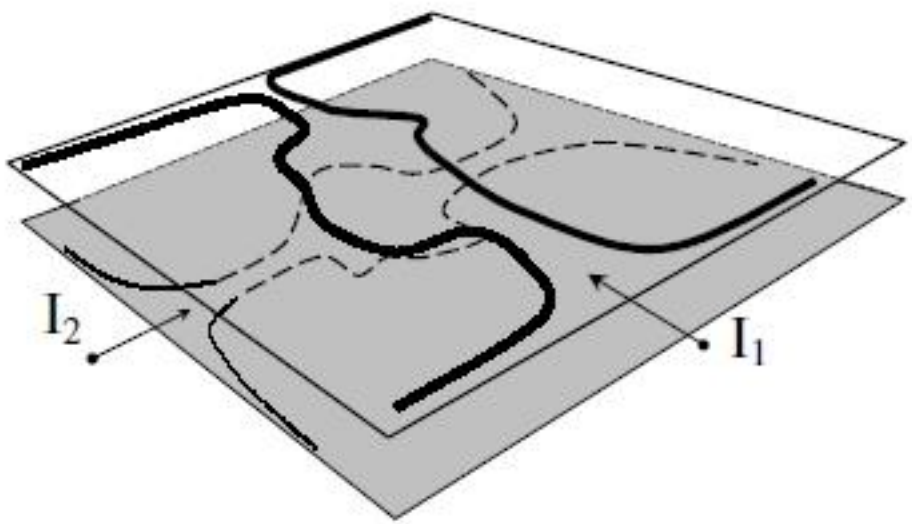
$$I_D(V) = \frac{e^3V^2}{h}\frac{\alpha'}{\Delta_2}\sum_n T_n(1-T_n)$$

Implies the view : Coulomb drag = hot spot physics

Noise induced transport

A. Levchenko and A. Kamenev, PRL 101, 216806 (2008)

Generic mesoscopic conductors



Conductance $g(E) = g + \delta g(E)$

Conductance fluctuations $\delta g(E)$

$g >> 1 \,,\ \delta g(E) \sim 1$

Thouless energy

$$E_T = \hbar D/L^2$$

Rectification

$$\Gamma(\omega) \sim \frac{e^3}{h}\frac{(\hbar\omega)^2}{E_T}$$

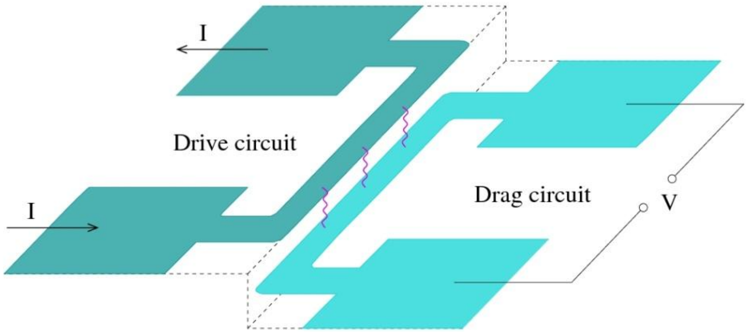
Drag current

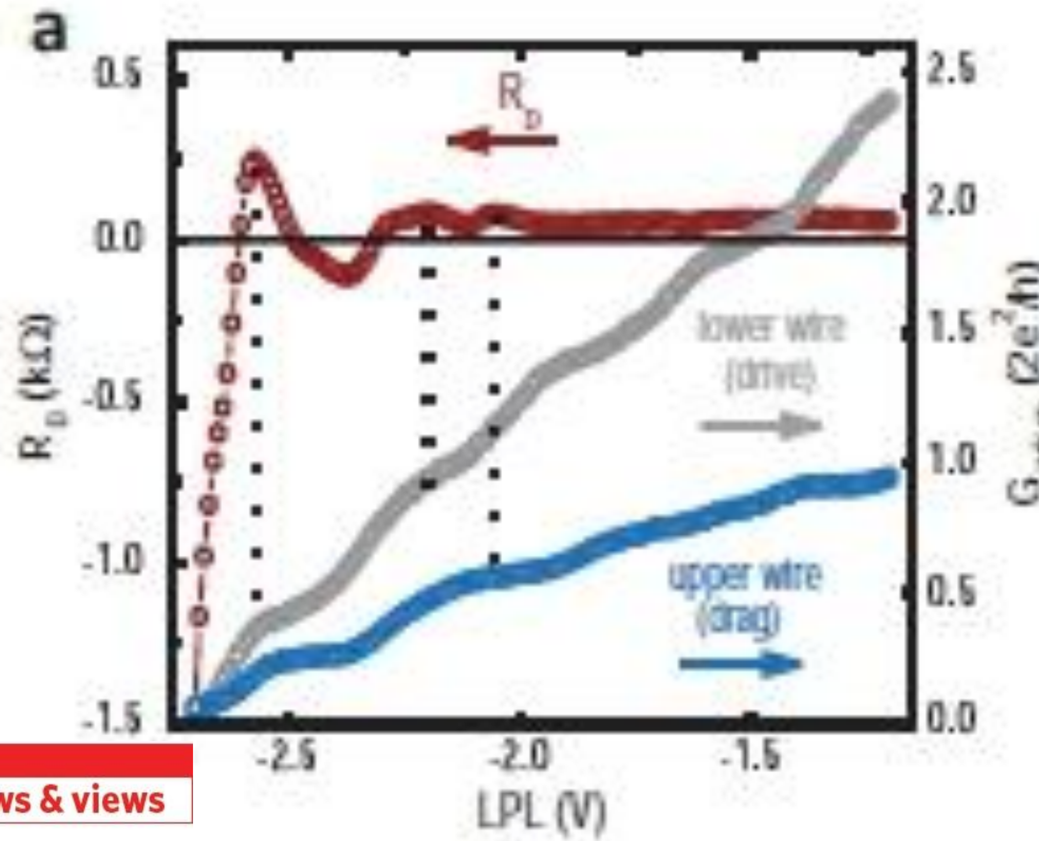
$$I_D \sim \frac{e^2 V}{h}\left(\alpha\frac{(kT)^2}{E_T^2} + \alpha'\frac{eV}{E_T}g\right)$$

External versus internal coupling : here charging of conductor neglected

McGill-Sandia Collaboration

Laroche, D., Gervais, G., Lilly, M. P. & Reno, J. L. ,
Nature Nanotech. ***6, 793–797 (2011).***





Buttiker, M., Sanchez, R,
Nature Nanotech. ***6, 757 (2011).***

news & views

NANOELECTRONICS

A closer look at charge drag

The observation that charges flowing through one quantum wire can drag charges in a second, unconnected wire either forwards or backwards requires a re-interpretation of Coulomb drag.

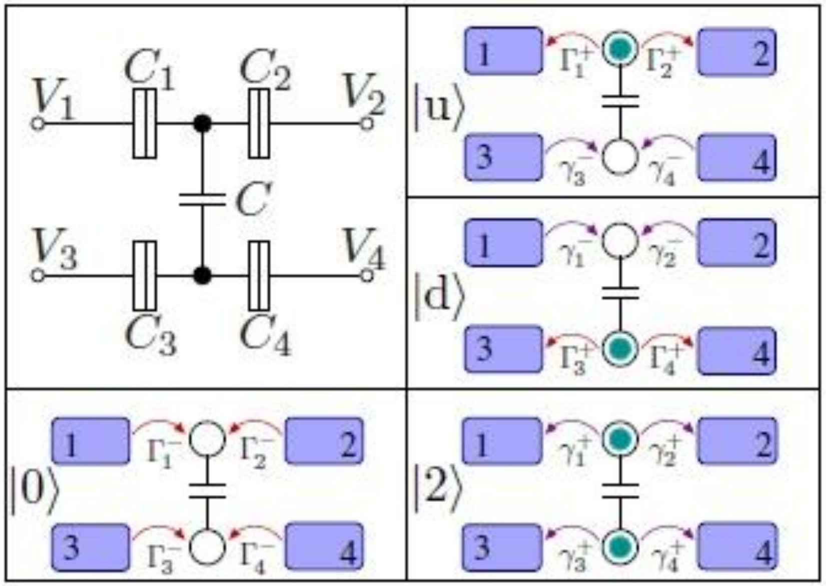
Markus Büttiker and Rafael Sánchez

(Shot)-Noise induced transport in quantum dots

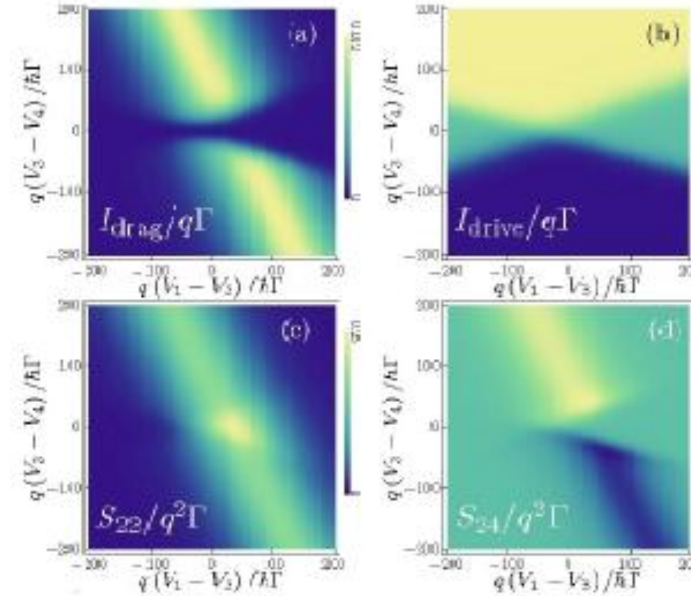
R. Sánchez, R. López, D. Sánchez, and M. Büttiker , Phys. Rev. Lett. **104**, 076801 (2010)

$V_1 = V_2$

$V_3 > V_4$

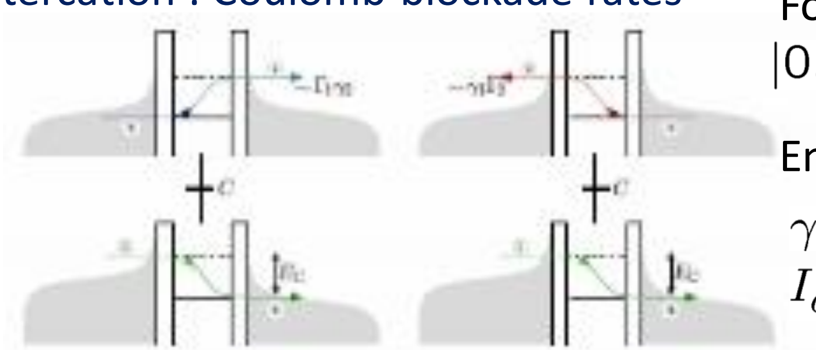


drive



gate

Realistic intercation : Coulomb blockade rates



Four states

$|0,0>, \ |0,1>, \ |1,0>, \ |1,1>$

Energy dependent rates

$$\gamma_1 \Gamma_2 \neq \Gamma_1 \gamma_2$$
$$I_{drag} \approx \gamma_1 \Gamma_2 - \Gamma_1 \gamma_2$$

Non-equilibrium noise (shot noise) of driven dot induces current in undriven dot

Fluctuation relations

D. Andrieux and P. Gaspard, J. Stat. Mech. (2007) P 02006

H. Forster and M. Buttiker, PRL 101, 136805 (2008)

Multiprobe (equal temperature)

$$P(N_1, N_2, ..., t) = P(N, t)$$

Generating function

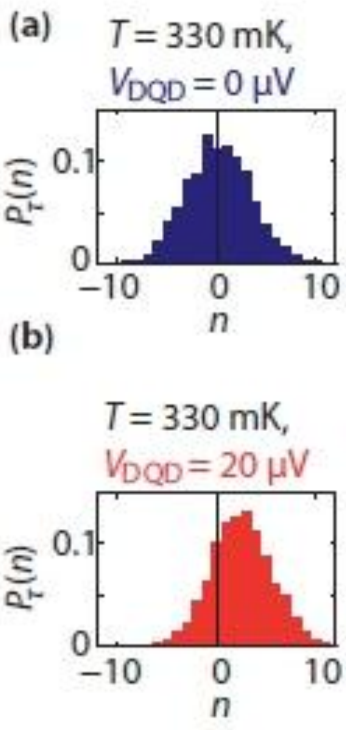
$$F(\chi) = ln \sum_N P(N, t) \exp(i\chi N)$$

Symmetry

$$F(i\chi) = F(-i\chi + qV/kT) \quad \Longrightarrow$$

$$P(N, t) = \exp(qNV/kT)\, P(-N, t)$$

Küng, Rössler, Beck, Marthaler, Golubev, Utsumi, Ihn, and Ensslin, Phys. Rev. X **2**, 011001 (2012)



Current

$$I_i = \sum_k G_{i,k} V_k + (1/2) \sum_{kl} G_{i,kl} V_k V_l + ...$$

Noise

$$S_{ij} = S_{ij}^{eq} + \sum_k S_{ij,k} V_k + ...$$

FDT

$$S_{ij}^{eq} = 2\, G_{i,j} kT$$

Gate voltage terminals

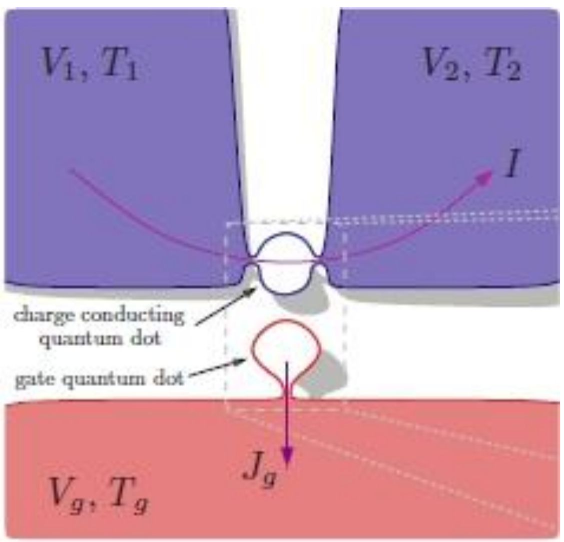
Carrier exchanging terminals

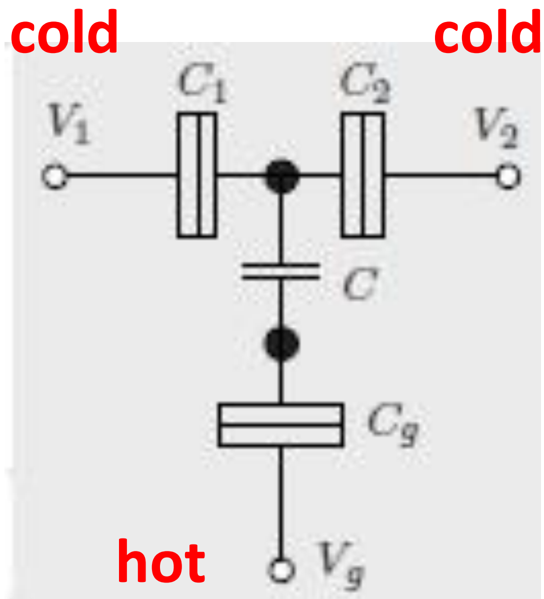
Noise suszeptibility and rectification

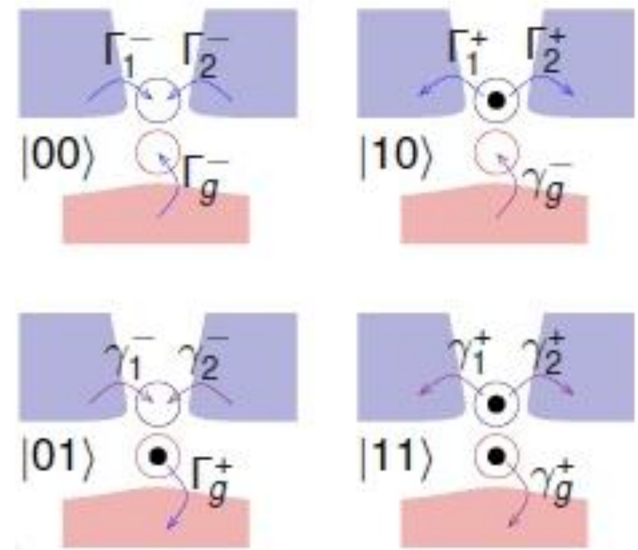
$$S_{ij,k} + S_{ik,j} + S_{jk,i} = kT\,(G_{i,jk} + G_{j,ik} + G_{k,ij})$$

Thermal-noise induced current

R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)







$$\Gamma_l^- = \Gamma_l\, f(\Delta_l^0/kT_l) \qquad \Gamma_l^+ = \Gamma_l\,(1 - f(\Delta_l^0/kT_l))$$

$$\gamma_l^- = \gamma_l\, f(\Delta_l^1/kT_l) \qquad \gamma_l^+ = \gamma_l\,(1 - f(\Delta_l^1/kT_l))$$

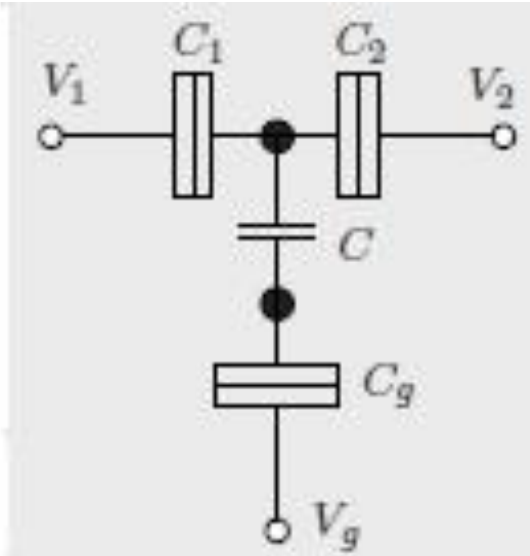
$$\Delta_l^n = \epsilon_u + U(1,n) - U(0,n) - qV_l$$

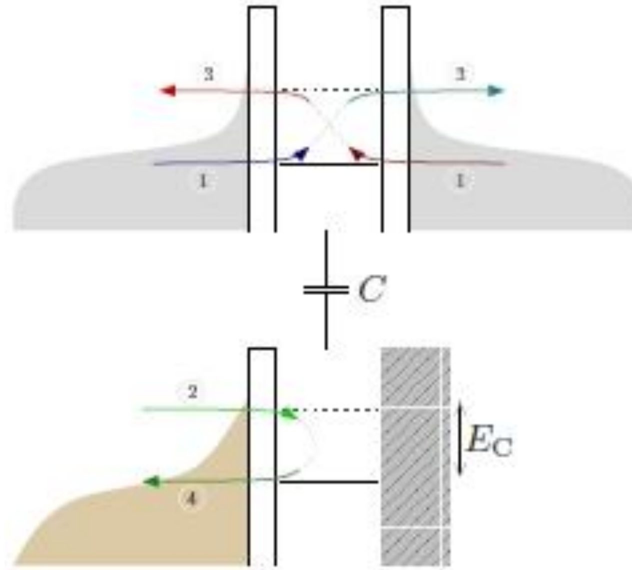
$$\Delta_g^n = \epsilon_d + U(n,1) - U(n,1) - qV_g$$

$$U(1,1) - U(0,1) = U(1,0) - U(0,0) + E_c \qquad E_c = \frac{2q^2C}{C_{\sum u}C_{\sum d} - C^2}$$

Heat to charge conversion

R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)





$$V_1 = V_2$$

$$I = q\,\frac{\gamma_1\,\Gamma_2 - \Gamma_1\,\gamma_2}{(\Gamma_1 + \Gamma_2)\,(\gamma_1 + \gamma_2)}\,\frac{J_g}{E_c}$$

If $\gamma_1 = 0\,,\ \Gamma_2 = 0 \implies$

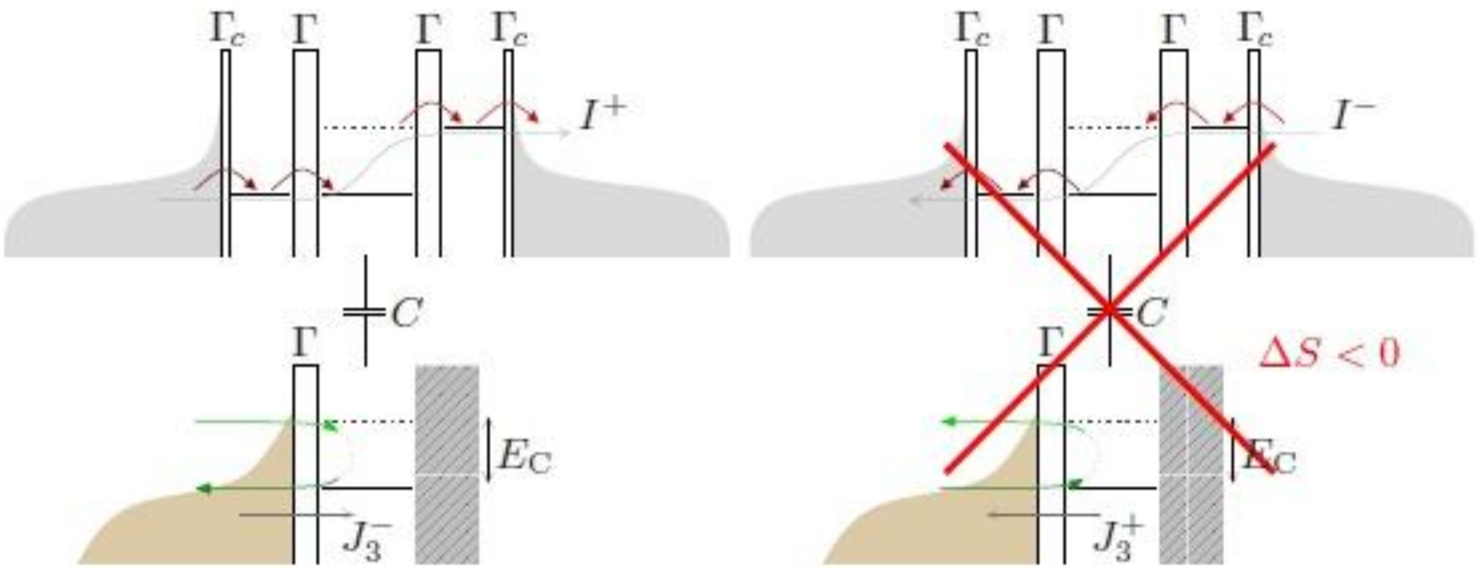
$$\frac{I}{q} = \frac{J_g}{E_c}$$

Every energy quantum of heat flow gets converted into a quantum of charge flow: optimal conversion

[T.E. Humphrey et al, Phys. Rev. Lett. 89, 116801 (2002)]

Optimal converter geometry

R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)



Power against the potential: $P = I\,(V_1 - V_2)$ $\quad I = qJ_g/E_c$

Efficiency $\eta = P/(-J_g) = q\,(V_1 - V_2)/E_c$

Stopping potential $\Delta S = 0\,,\; I^+ = I^-\,,\quad \eta_C = 1 - T_s/T_g = q\Delta V_*/E_c$

(reversibility)

Efficiency at maximum power

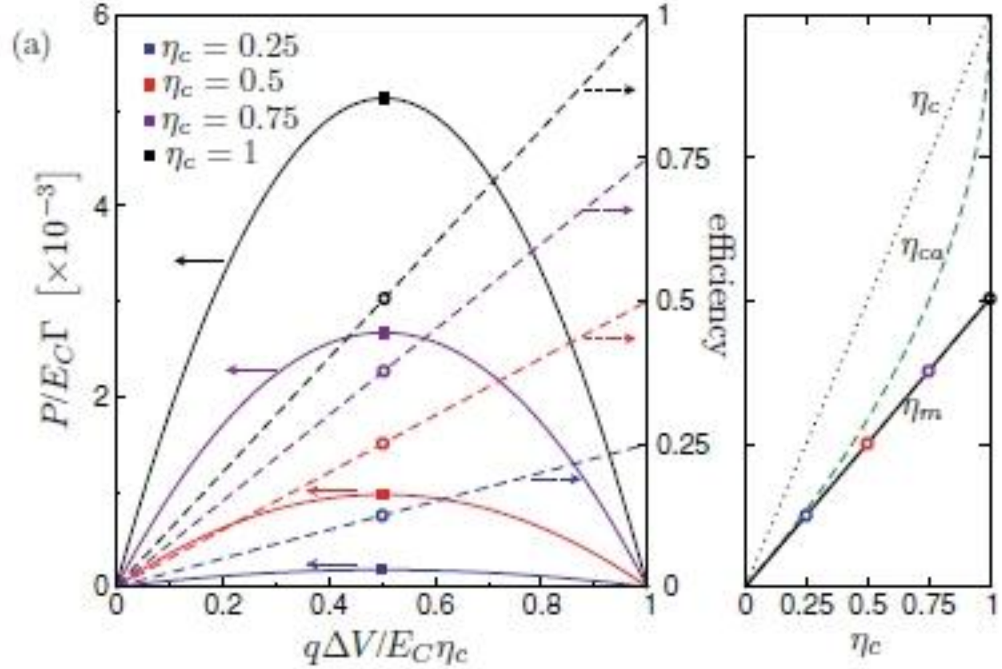
R. Sanchez and M. Buttiker, PRB 83, 085428 (2011)

Carnot efficiency (no power) $\eta_C = 1 - T_s/T_g$

Curzon-Ahlborn efficiency $\eta_{ca} = 1 - \sqrt{T_s/T_g}$

Efficiency at maximum power η_m

Power of noise induced current as a function of load potential $\triangle V$
(units stopping potential)



$$q\Delta V/E_C\eta_C = \Delta V/\Delta V_*$$

$$\eta = P/(-J_g) = q\,\Delta V/E_C = \eta_C\,\Delta V/\Delta V_*$$

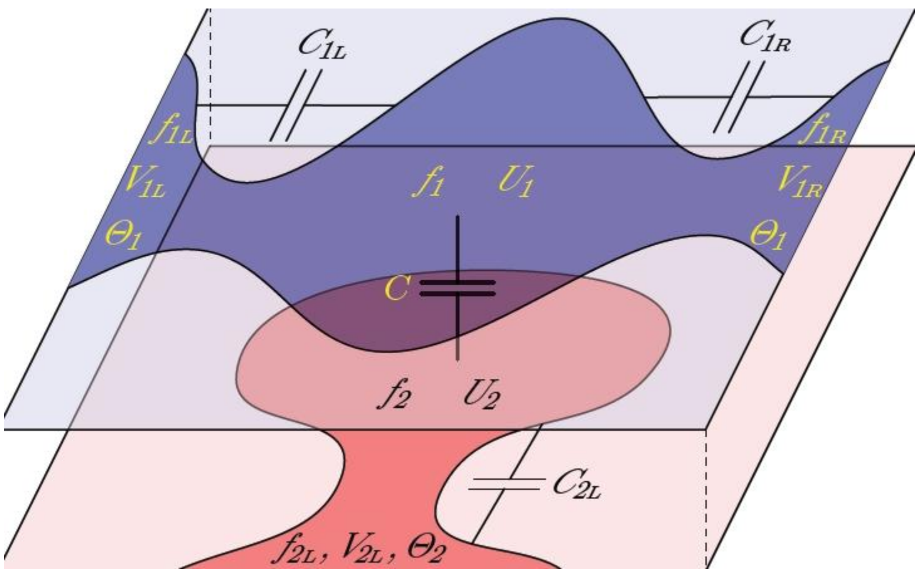
Current from open dots

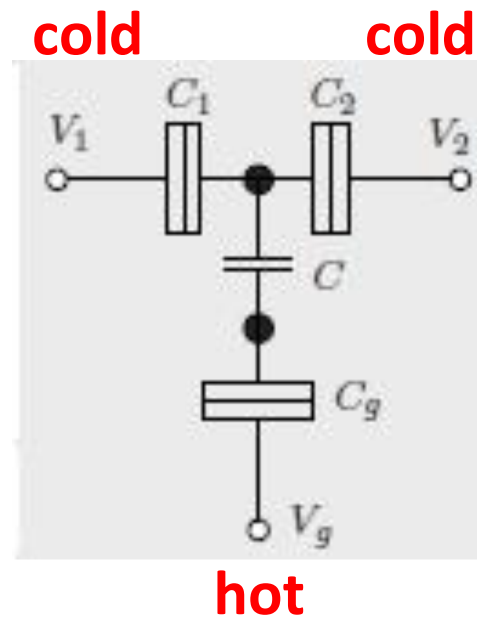
B. Sothmann, R. Sanchez, A. N. Jordan, M. Buttiker, Phys. Rev. B **85**, 205301

Coulomb blockaded dots generate very small currents

Can we get larger currents in open dots?

Levchenko and Kamenev: Yes, but at best a single quantum channel effect.





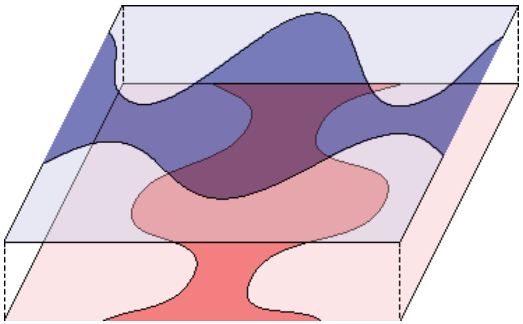
$$e\nu_i \frac{df_i}{dt} = e\nu_i \frac{\partial f_i}{\partial U_i}\dot{U}_i + \frac{e}{h}\sum_r T_{ir}(f_{ir} - f_i) + \delta i_\Sigma,$$

$$f_i(E, dU_i)\,,\ \ \delta i_\Sigma(f_i(E, dU_i))$$

Noise also depends on potentials

Current from hot spots: open dot

B. Sothmann, R. Sanchez, A. N. Jordan, M. Buttiker , Phys. Rev. B **85**, 205301



Change figure to one lead

For $C_{eff} = 10fF$

$G' = (e^2/h)(mV)^{-1}$

$\Theta_1 - \Theta_2 = 1K$

the resulting current is 0.1nA

Efficency at maximum power

$$\eta_{max} = \frac{1}{4\tau_{RC} G_1} \Lambda^2 k_B(\Theta_1 - \Theta_2)$$

Asymmetry

$$\Lambda = \frac{G'_{1L} G_{1R} - G'_{1R} G_{1L}}{G^2_{1\Sigma}},$$

Noise induced dc-current

$$\langle I_{1L} \rangle = \frac{\Lambda}{\tau_{RC}} k_B(\Theta_1 - \Theta_2)$$

Effective RC-time

$$\tau_{RC} = C_{eff}/G_{eff}$$

$$G_{eff} = G_{1\Sigma} G_{2\Sigma}/(G_{1\Sigma} + G_{2\Sigma})$$

$$C_{eff} = ...$$

Stopping voltage

$$V_{stop} = \Lambda k_B(\Theta_1 - \Theta_2)/(G_1 \tau_{RC})$$

Maximum power

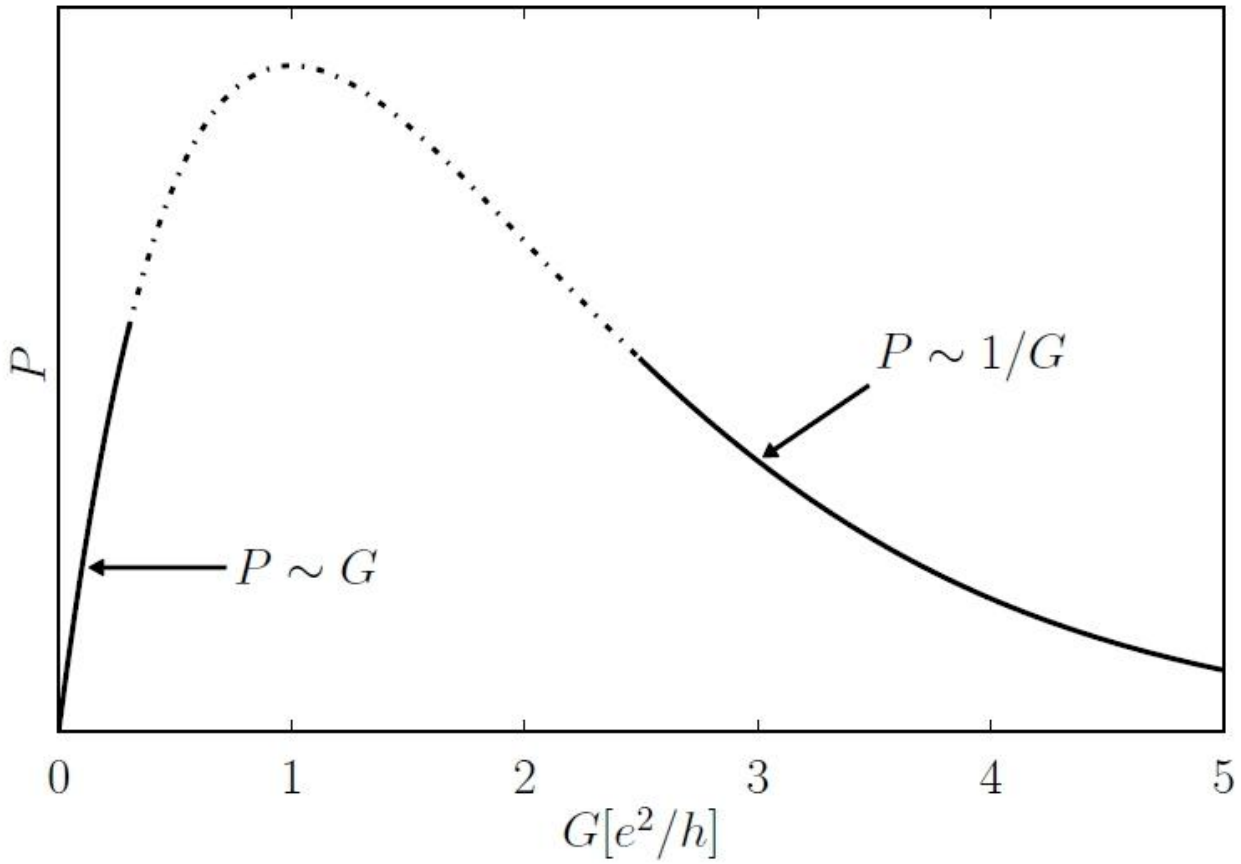
$$P_{max} = \frac{\Lambda^2}{4\tau^2_{RC} G_1}(k_B(\Theta_1 - \Theta_2))^2$$

Intercavity heat current

$$J_H = \frac{1}{\tau_{RC}} k_B(\Theta_1 - \Theta_2)$$

Power quantum dot energy harvesters

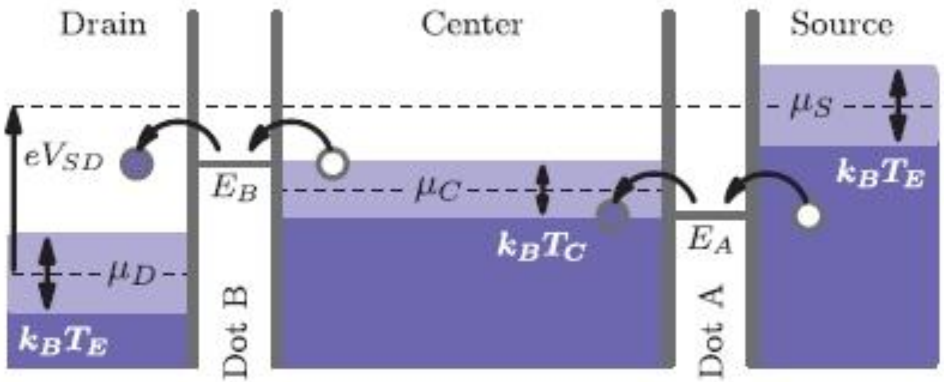
B. Sothmann and M. Buttiker, (unpublished).

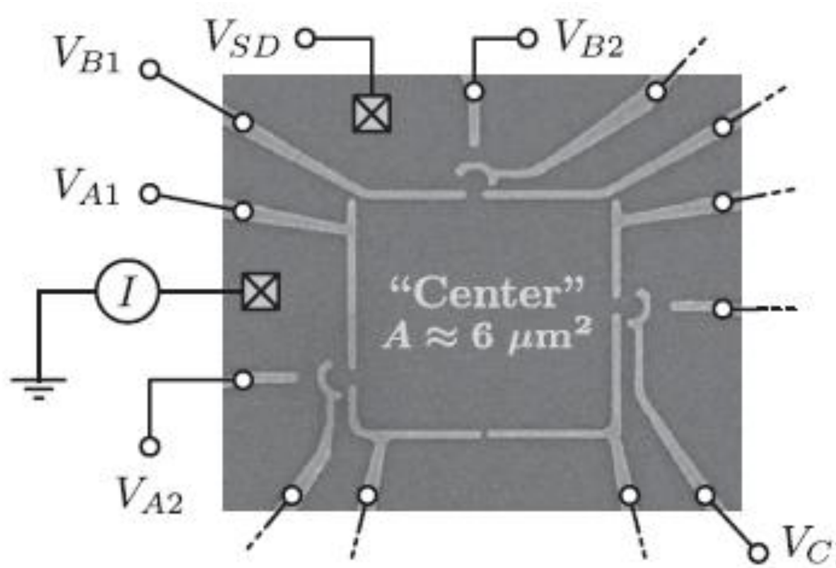


Maximum power geometry

Discussed as a refrigerator:

J. R. Prance*, C. G. Smith, J. P. Griffiths, S. J. Chorley, D. Anderson, G. A. C. Jones, I. Farrer, and D. A. Ritchie , Phys. Rev. Lett. 102, 146602 (2009)





Resonant levels provide contacts with conductance

$$G = 2\frac{e^2}{h}$$

The measurement problem

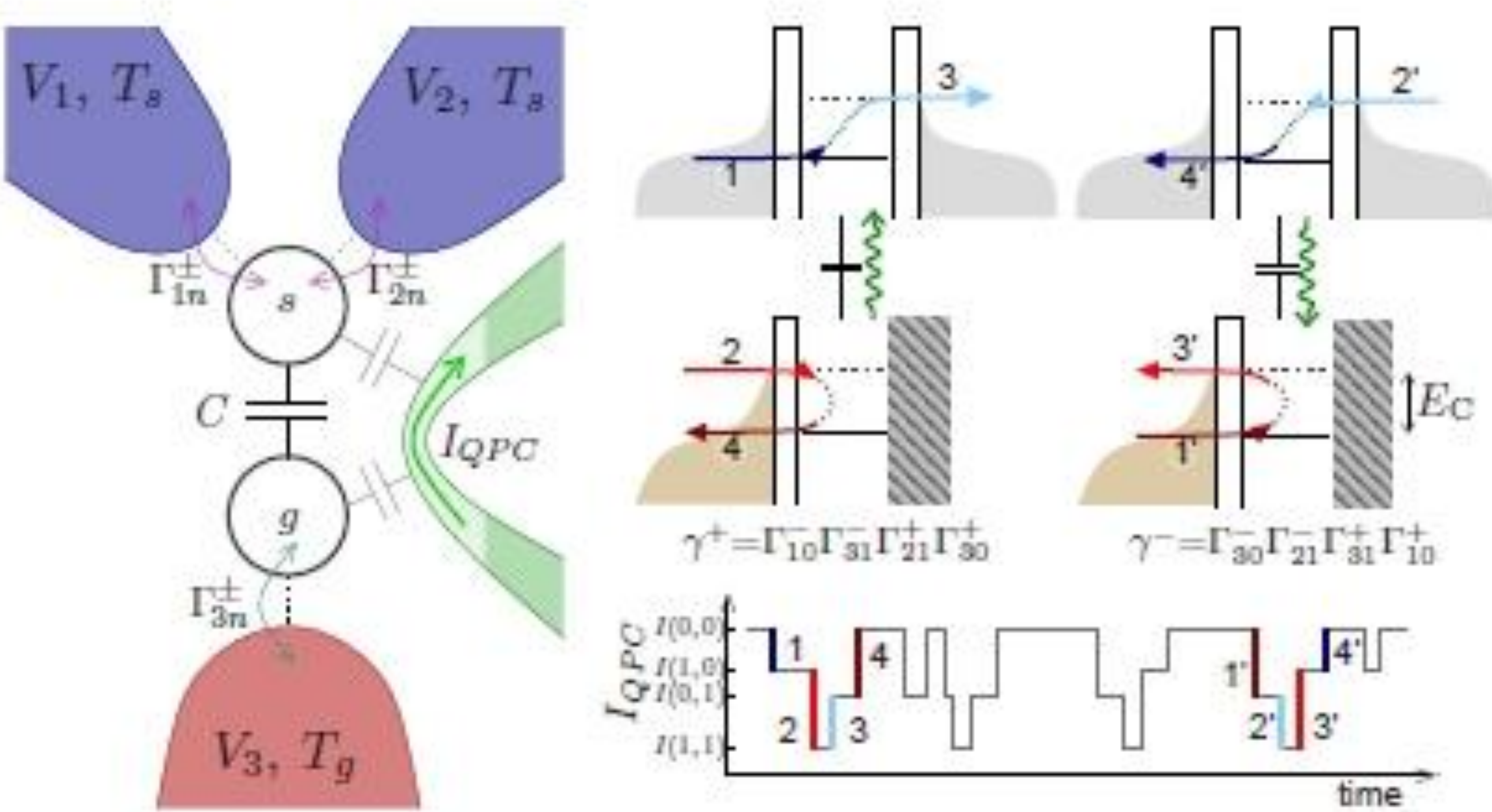
Rafel Sanchez and Markus Buttiker, (unpublished)

Contrary to charge currents, energy or heat currents are difficult to measure in Nano-scale and mesoscopic structures . Typically a conversion to an electrical signal is needed. For instance the Seebeck coefficient gives a voltage in response to a temperature gradient. The voltage is easy to measure but even just to measure the temperatures difference which acts as thermoynamic force in a small scale structure is difficult.

In the following we return to the two quantum dot problem and discuss an electrical measurement proceedure for energy and heat currents

Heat flux detection

Rafel Sanchez and Markus Buttiker, (unpublished)



State resolved fluctuation relations

In the presence of reservoirs at different temperatures fluctuation relations invoke both Charge and heat currents. Alternatively it is possible to write fluctuation relations for state resolved counting.

N_{ln} number of charges transferred through terminal l with system in state n

ζ_{ln} State resolved counting field conjugate to N_{ln}

$\tilde{A}_{l,n} = \Theta^H_{l,n}\beta_l$ State resolved affinity

$\Theta^H_{l,n} = E_{\alpha,n}(\{V_m\}) - qV_l$

Incomplete fluctuation relation (affinities are taken at different occupations)

$$\mathcal{F}(i\zeta_{sn}) = \mathcal{F}(-i\zeta_{sn} - \tilde{A}_{sn} - \tilde{A}_{g\bar{n}})$$

In terms of charge current I(system) and heat current (gate)

$$\frac{1}{t}\ln\frac{P(I_{sn})}{P(-I_{sn})} = I_C\,(V_1 - V_2)\,\beta_s - I_{H,g}(\beta_g - \beta_s)/2$$

Depends on state resolved currents $I_{H,g} = 2E_C(I_C - I_{11} + I_{20})$

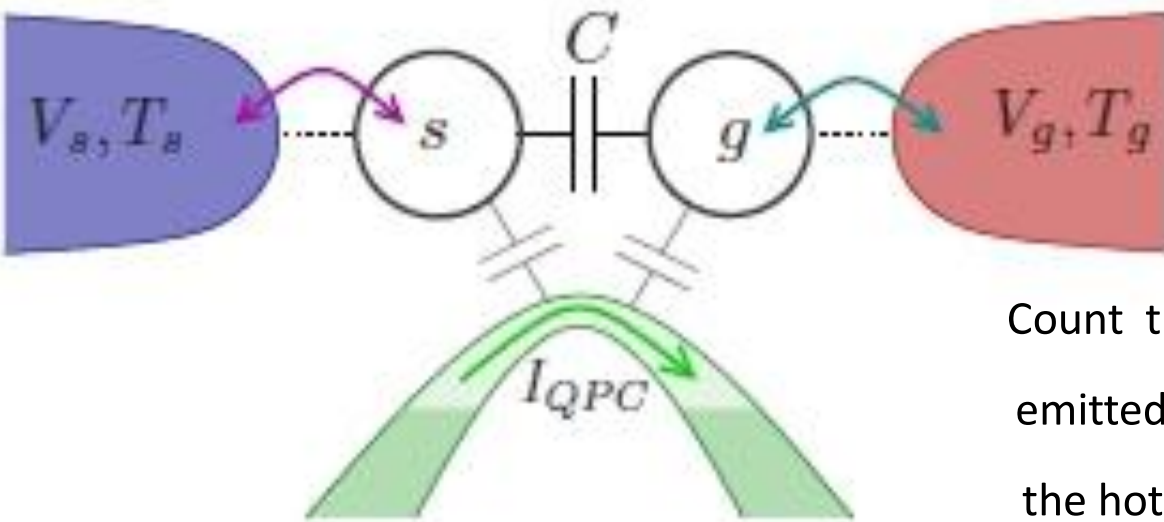
Heat flux in diode configuration

T. Ruokola and T. Ojanen, Phys. Rev. B 83, 241404 (2911)

R. Sanchez and M. Buttiker, (unpublished).

Geometry with heat transport only

Only Coulonb energy Ec is exchanged :



Count the number of electrons N_{sn}

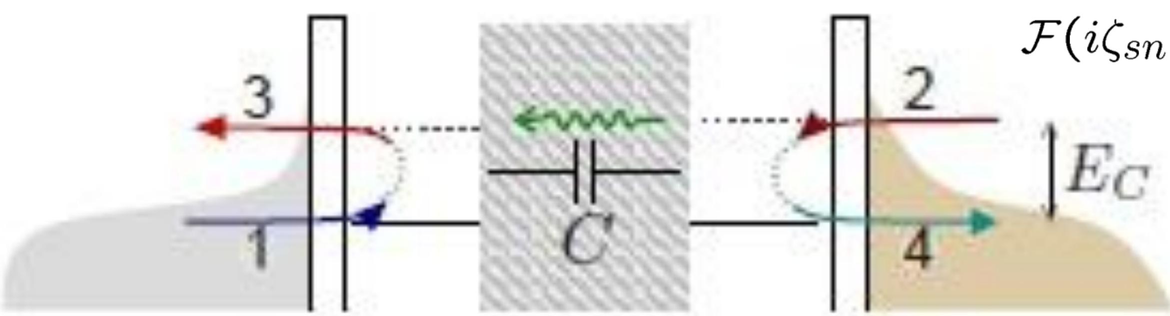
emitted to the cold system conditioned on

the hot dot containing n electrons

Cumulant generating function

$\mathcal{F}(i\zeta_{sn})=\mathcal{F}(-i\zeta_{sn}+E_C\,(\beta_g-\beta_s))$

Fluctuation theorem for heat



Summary

Three-terminal thermoelectrics

Current generated from non-equilibrium noise (hot spot)

Coulomb coupled quantum dots

Direction of heat current decoupoled from direction of charge current

Optimal heat to charge conversion

One energy quantum of the bath transfers one quantum of charge

System provides a good test of non-equilibrium fluctuation relations

Quasi-classical dot (open contacts)

Determination of heat flows through charge detection